OPTIMAL DESIGN OF WATER DISTRIBUTION SYSTEM

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CERTIFICATE

Certified that the work presented in this thesis entitled 'Optimal Design of Water Distribution System' by Shri Rajendra Singh Solanki has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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OPTIMAL DESIGN OF WATER DISTRIBUTION SYSTEM

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Mpst of the methods developed so far for the optimal design of looped water distribution systems employ nonlinear optimization techniques along with a network solver. Nonlinear optimization techniques require a high degree of experience in applying these to complex problems in order to give initial values of various parameters. It becomes very difficult to solve the nonlinear problem when measures are taken to account for the variable demand pattern and discrete nature of some of the variables (e.g. pipe diameters).

The present work is aimed at developing a computer program based on the Linear Programming Gradient (LPG) method as proposed by Alperovits and Shamir. No network solver is required in this approach. The program developed here deals with the problem of designing an optimal water distribution system for a single loading pattern and gives the solution in terms of the available pipe diameters, head at the source (elevation of the reservoir or the horse-power of the pump), and capacity of the booster pump if required at a specified location.

1. INTRODUCTION

Water is a primary necessity of life. Throughout recorded history large cities have been concerned with their water supplies. Even ancient cities found that local sources of supply - shallow wells, springs, and brooks - were inadequate to meet the very modest sanitary demands, and the inhabitants were constrained to build aqueducts which could bring water from distant sources. In the present times, the need of supplying treated water to community is well recognized. Extensive distribution systems are needed to deliver water to the individual consumers.

A water distribution system is meant for supplying water at sufficient pressure and in adequate quantity to various points in a locality. The distribution system consists of various components such as pipes, reservoirs, pumps, and valves. An engineer is concerned with the design, operation and maintenance of a distribution system i.e. deciding about the sizes or capacities of different components in the system and their operating policies.

There are some basic rules forming the foundation of all design procedures for a water distribution network, such as flow continuity at the nodes, head continuity along paths of the network, and the head loss vs. flow relationships for different components in the system.

In the conventional design procedures, tentative sizes of components in the system are assumed and then the analysis techniques are applied to check the feasibility of the system regarding the required amount of water being supplied at adequate pressures to all nodes of the network. Hence, a number of combinations of the component sizes could be possible to serve the same purpose. Optimization techniques are very useful in selecting the best alternative out of a number of possible solutions in view of the constraints on the resources and the objective to be achieved.

Considerable saving in cost can be achieved by using the optimization techniques over the conventional methods of water distribution system design and operation. This water distribution system is often the major investment of a municipal waterworks. Such saving in cost is, therefore, of immediate concern as India being one of the participating countries at the World Water Conference held in 1977 at Mar del Plata, Argentina, has committed itself to the drinking water supply and sanitation programme during the decade 1981-1990. No citizen of this country should be denied access to safe potable water by the year 1990. This would involve investments well over Rs. 15,000 crores. Development of efficient techniques of design could go a long way towards the fulfilment of a programme of such a vast magnitude.

1.1 Objective of the Study

Various investigators have developed a number of optimization algorithms for designing a water distribution system. Most of methods proposed utilize nonlinear optimization techniques for looped network systems. However, recently some work has been done in the area of application of linear optimization techniques for designing a looped network[2]. Application of linear programming may be promising in view of its being more efficient compared to nonlinear programming.

The present work is aimed at developing a computer programme based on 'Linear Programming Gradient' method developed by Shamir and testing it on a few networks of varying sizes. The various cost data taken are in accordance with the local conditions.

2. LITERATURE REVIEW

2.1 Mathematical Models for Steady State Analysis

Water distribution systems are designed to deliver water from sources to consumers in the required quantity and under a satisfactory pressure through pipeline networks equipped with a variety of components such as pumps, valves of various types, distribution reservoirs. The mathematical model of a network consists of links connected at nodes. Nodes are points at which two elements connect, or at which a flow enters or leaves the network. To maintain mathematical tractability it is customary to model only the major structure using some appropriate representation of the detailed structure of the actual system. For example, it is customary to include in the model only pipes above some minimum diameters, and also to lump withdrawls taken along the pipes and assign them to the nodes.

At each node we define the head and the consumption. Head is the sum of the topographic elevation, the pressure and velocity heads (the latter is usually negligible) While there exists a head at any point along a pipe we shall refer by "heads" to the set of values at the nodes. At certain nodes there are imputs of water into the system. At others there are consumptions, which are viewed as negative inputs.

Each link is characterized by a physical law which relates the flow through it to the head difference between its ends. For pipes there are several empirical flow equations; one commonly used is the Hazen-Williams equation:

$$Q = \alpha C_{HW} D^{2.63} (\Delta H/L)^{0.54} \tag{1}$$

where Q is the discharge; C_{HW} a smoothness coefficient (the Hazen-Williams coefficient); D the diameter; ΔH the head difference between the two end nodes, and L the length, α is a numerical coefficient, whose value depends on the units used (For Q in m^3/sec and D in cm $\alpha = 5.4 \times 10^{-3}$; for Q in cfs and D in inches $\alpha = 6.28 \times 10^{-4}$). $\Delta H/L$ is the hydraulic gradient (hereafter denoted by J).

For centrifugal pumps, the head added by the pump, ΔH, is usually approximated by a polynomial of the form

$$\Delta H = a + bQ + cQ^2 \tag{2}$$

Every other type of element has its own law. Reservoirs are connected at nodes; water level in the reservoir is the head at that node.

The mathematical model of the network is a set of simultaneous non-linear algebraic equations, which correspond to a steady state flow in the network under fixed boundary conditions: a set of inflows and consumptions at the nodes and a set of fixed heads at specified nodes. The solution of these equations is called a steady state

flow solution. There are two basic types of equations: node equations and path equations. The first express material continuity at a node. For node j:

$$\sum_i Q_{ij} + I_j = 0 \tag{3}$$

where Q_{ij} is the flow from node i to node j and I_j is the inflow into node j. Consumptions (withdrawls) are denoted by C_j and appear as negative values of I_j.

Path equations equate the head difference, b_p, between the end nodes of path (p) in the network to the sum of head gains and losses in all links belonging to this path:

$$\sum_{i,j \in p} \Delta H_{ij} = b_p \tag{4}$$

A path may connect any two nodes. Usually path equations are formulated between pairs of nodes at which the heads are known. A special case are loop equations, where the two ends are at the same node, and then $b_p = 0$.

There are many different ways to construct the mathematical model. In a network with N nodes, a set of N node equations fully determines the flow solution, provided one head (a reference head) is given. When loop equations are used one needs as many equations as there are basic loops, i.e., loops which do not have pipes intersecting them. General path equations can be formulated in many ways and the rule is that they should be mutually independent. Any one of these formulations can be used to

obtain the flow solution i.e. the heads and flows throughout the network. The set of simultaneous non-linear algebraic equations can be solved by an appropriate iterative technique - Hardy-Cross, Newton-Raphson, or linearization[35,36,37].

When there is internal storage in the system, i.e. there are operational reservoirs then the sequence of flow solutions for varying withdrawls over time (e.g. a day) is of importance. Subsequent flow solutions are linked through the changes in water levels in the reservoirs. Simultation of such a system's operation is carried out by obtaining a flow solution for the initial boundary conditions, the resulting flows are used to update reservoir levels, then these levels are used as boundary conditions for the next flow solution, and so on[9,31,35,36].

2.2 Problems of Planning, Design and Operation

The layout of the small-diameter pipes in a distribution network is essentially fixed by the land development. Water has to be delivered to every building and pipes usually follow streets, so the layout of the finer grid is fixed to a large extent. On the other hand, the engineer has to select the pipe materials and diameters, consider the possibility of breaking network into pressure zones (separated by special pressure regulating devices), allocate the loads to the various sources, determine the layout and sizing of the feeder mains, and fix the

locations and designs of the pumps, reservoirs and other facilities. Planning is the phase of selecting the layout and main features of the system. Design is the phase of fixing sizes and characteristics of the various components. The two are complementary and should be carried out simultaneously.

Distribution systems have to operate under time-varying conditions. Considering long range changes, on the scale of years, there is the need to meet increasing demands. This capacity expansion problem is common to many areas of engineering. The scale of time-varying conditions which is of specific concern is associated with the operation of the system. Water distribution systems operate under loads which change over the day, the week and the seasons. Sources, pumps, valves and reservoirs are operated to meet these varying demands. Setting the operating policy of the system is an integral part of the design process, since sizing the components depends on their operation and vice versa.

In the design one would like to consider explicitly the detailed operation of the system, that is, the hour by hour position of the pumps, valves and reservoirs. No way has been found as yet to formulate and solve the design and optimization problem in this form. Instead one can include in the formulation of the design problem specification of the operation under one or several loadings (sets of consumptions). These are "typical" or "critical"

loadings - for example, average daily consumptions, maximum demands during the day, low demands (which usually occur at night, during which reservoirs can be filled), high demands for fire fighting (usually concentrated in the highly congested business district), etc. The solution of this design and operation problem provides both sizing of the various components and their operation under these typical loadings.

Planning, design and operation are thus three aspects of a single problem, but under certain circumstances the task may be narrowed to design and operation, or operation alone. In the following section which deals with optimization, we shall treat planning and design as a single problem. The method is as follows: one specifies facilities (pipes, pumps, valves, reservoirs) wherever they seem reasonable; the optimization procedure is allowed to set any design variable to zero, thereby eliminating the element from the solution. This procedure will not create a facility where one was not specified, and is therefore limited to those configurations stipulated by the designer. Still, this procedure allows for selection among alternatives.

2.3 Formulation of Network Optimization Models

We shall first discuss the general structure of optimization problems for planning design and operation of water distribution systems and then go on to describe and discuss the methods which have been developed to solve them.

2.3.1 Decision Variables

In selecting pipes one has to decide on their material, diameter and wall thickness (pressure bearing capacity). In the optimization, it is usually assumed that material (i.e. smoothness) and wall thickness have been fixed, and the remaining decision is on the diameter. In selecting pumps there are various considerations, but for the optimization only the head vs. discharge characteristics of each pump will be considered as the decision variable. Similar considerations hold for valves and reservoirs.

Decision variables of the planning and design problem include:

a) Pipe diameters,

b) Pump locations and characteristics,

c) Valve locations, and

d) Reservoir locations and sizes.

When operation of the system under a set of typical or critical loadings is included in the formulation, the following decision variables are added:

e) Pumps to be operated (ON/OFF) under each loading, and

f) Valves to be operated (amount of pressure loss provided by the valve) under each loading.

2.3.2 Objective Functions

Minimum cost is the criterion most often used in optimization of water distribution systems. Total cost

is made of capital plus operating costs. The latter usually reflects only energy costs, since operation and maintenance may be included in the capital cost.

Performance indicators, such as minimum pressures at supply nodes, are usually treated as constraints in the optimization. Some work has been done in which certain performance criteria were used in the objective functions in a multiple-objective formulation[10].

2.3.3 Constraints

Several types of constraints appear in the optimization models. First, the physical laws of flow in the network have to be satisfied. These are equality constraints for continuity of mass and/or of hydraulic head lines, equations (3) and/or (4). The consumptions are usually treated as fixed externally, so they appear as constraints in the node continuity equations. Limits are normally set for the heads or pressures at some nodes. Minimum pressures are to be guaranteed under all loadings, to meet service standards for domestic and industrial consumers and to ensure sufficient operating conditions for fire fighting. Maximum pressures are specified when there is a danger of pipes bursting or equipment being damaged under excessive pressure.

Special types of constraints may arise from specific formulations of the optimization models. These will be discussed specifically for each model to be presented.

2.4 Branching Network Models

2.4.1 Linear Programming Model

Considering a branching network, supplied from one or more sources by gravity, and designed for a single loading, having specified consumptions at the supply nodes to be satisfied (C_j at node j); at some or all of the nodes the head, H_j, is to be within a given range, $HMIN_j$ to $HMAX_j$. The layout is given, and the length of the link connecting nodes i and j is L_{ij}.

The linear programming design procedure[16,17,19,24] is based on a special selection of the decision variables: for each link allow a set of "candidate diameters", the decision variables being the lengths of the segments of these diameters within the link. Denoting by X_{ijm} the length of the pipe segment of the m-th diameter in the link between nodes i and j, then

$$\sum_{m} X_{ijm} = L_{ij} \qquad \text{for all } (i,j) \tag{5}$$

where each link may have a different set of candidate diameters. For a branching network in which the consumptions are known, the discharges in all links, Q_{ij} are fixed. The head loss in segment m of the link is:

$$\Delta H_{ijm} = J_{ijm} X_{ijm} \qquad \text{for all } (i,j,m) \tag{6}$$

where J is the hydraulic gradient ($\Delta H/L$ in eq. (1)) which is a function of the discharge and of the diameter (if the

smoothness is assumed to be selected in advance, and therefore fixed).

Starting from any node in the system, s, at which the head is fixed (e.g. a reservoir), and selecting any path from it to node n, at which the head has to be within a given range, one may formulate the constraint

$$HMIN_n \leqslant H_s \pm \sum_{(i,j)} \sum_m J_{ijm} X_{ijm} \leqslant HMAX_n \qquad (7)$$

The first summation is over all links along the selected path, and the second over all segments of the link. The signs of the terms depend on the direction of flow in the link. In order to reduce the number of constraints and improve computational efficiency, head constraints may be formulated only for part of the nodes, provided they suffice to ensure that pressures throughout the network are within their acceptable ranges. If this method is used one has to examine the solution to ascertain that all heads are satisfactory; wherever they are not, a new constraint has to be added and the problem resolved.

The cost of a pipeline with a fixed diameter can reasonably be taken as linearly proportional to its length. Thus, the total cost of the pipeline network is

$$\sum_{(i,j)} \sum_m c_{ijm} X_{ijm} \qquad (8)$$

Minimization of (8), subject to (5), (7) and non-negativity of the X is a linear program (LP).

It can be shown that if the cost of a pipe is a convex function of the diameter (as it normally is) then in the optimal solution of the LP each link will contain at most two segments, their diameters being adjacent on the candidate list for that link[2].

The LP formulation can be extended to include the cost of pumps, and their operation, using linear or linearized cost functions[2,24]. Reservoirs can also be included, using the head in the reservoir as the decision variable, and fitting it with a linear cost function. More than one loading can be considered. Each loading results in a set of constraints of type (7), possibly with different bounds on the heads for each loading, and the entire set is solved simultaneously in the LP. If energy costs are included the objective function contains a weighted sum of the energy costs for operating under the different loadings.

2.4.2 Dynamic Programming Model

Optimal design of a branching network, with or without pumps and reservoirs, can easily be formulated as a dynamic programming (DP) problem. The solution requires more computer time than the LP method, but the formulation is free of certain shortcomings present in the LP.

Kally[21] used DP to optimize the diameter and wall thickness (which determines the pressure bearing capacity) of the segments of a pipeline, as well as the heads added by the pump located along it. The objective

function included capital plus operating costs. Similarly, Liang[28] used DP to optimize the diameters of segments between takeoff points to consumers along a pipeline fed by a pump at its upstream end. The objective was to minimize capital cost, subject to minimum pressure constraints at the takeoffs.

Probably because of its computational inferiority, the use of DP for branching networks has not been developed to an operational stage. Still, for completeness, the structure of DP formulation is presented here.

The network is divided into segments. A segment is defined between adjacent takeoff points, so that the discharge remains the same along the segment, or, if these segments are too long, a finer division may be used, allowing pipe properties to change from one segment to the next. Takeoffs are given, and minimum pressures are to be satisfied at each takeoff node. Decision variables may include the diameters, material (roughness and strength) and class (pressure bearing capacity) of pipes, and the capacities of pumps. For clarity of the presentation we shall assume that material and class of the pipes have been fixed, so that diameters are the only decision variables. The objective function may include capital cost of pipes and pumps, energy costs, and any benefits, costs or penalties which are the functions of the heads at the nodes. No assumptions, such as continuity or convexity, have to be

made about these functions. The quantities to be supplied at nodes are assumed fixed, and constraints may be imposed on the minimum and/or maximum heads at the nodes.

The state variables are the heads at the nodes, and the nodes are the stages. Computation proceeds upstream, starting from the downstream end of each branch. The recursive equation of the DP is :

$$F^*_{j+1}(H_{j+1}) = \underset{D_k}{\text{Min}} \left[g(D_k) + f(H_{j+1}, H_j) + F^*_j(H_j) \right] \qquad (9)$$

D_k is the diameter of segment k which connects node (j+1) to its downstream neighbour, node j. $g(D_k)$ is the cost of this segment. H_{j+1} is the head at node (j+1); H_j is the head at node j and given H_{j+1} and the diameter D_k, it ~~and~~ can easily be computed since the discharge in the segment is known $f(H_{j+1}, H_j)$ is the cost (or benefit) associated with the link, for the given heads at its two ends. $F^*_j(H_j)$ is the optimal value for the portion of the system downstream of node j, given the head at node j. The minimization is over all admissible values of D_k, and is performed for each of a set of discretized values of H_{j+1} over its admissible range. Whenever $H_j = H_j(H_{j+1}, D_k, Q_k)$ is outside its admissible range, the examined D_k is disallowed.

For a pump located between nodes (j+1) and j one uses:

$$F^*_{j+1}(H_{j+1}) = \underset{H_j}{\text{Min}} \left[g(H_{j+1}, H_j) + f(H_{j+1}, H_j) + F^*_j(H_j) \right] \qquad (10)$$

$g(H_{j+1}, H_j)$ here is the capital plus operating cost for a pump designed to deliver the known discharge, Q_k, from head H_{j+1} at its intake to head $H_j($ $H_{j+1})$ at its discharge. The minimization is over a set of discrete values of H_j.

At every branching node of the network one adds up the values of F^* for the downstream branches which connect at it. At such a node j, $F_j^*(H_j)$ is still the optimal cost of the part of the network downstream from it, except that now it is a sum of the optimal costs for all branches originating at node j. Thus, one follows the single line procedure outlined above, starting from all downstream extremities of the network.

2.5 Looped Network Models

2.5.1 Linear Programming Models

Lai and Schaake[26] have attempted this problem by making the assumption that the heads at all nodes, as well as the demands are given in advance. The solution thus gives the optimal diameters for the assumed pressure pattern. Since the heads are fixed, the flow through each link is a function of the link properties only, and if the length and pipe material are given, the flow is a function only of the diameter. For node j at which a demand C_j has to be satisfied, the following constraint is a statement of continuity at the node:

$$\sum_i Q_{ij} = \sum_i K_{ij} D_{ij}^{2.63} = C_j \qquad (11)$$

where K_i is a coefficient whose value is determined by the given data: the heads at nodes i and j, and the length and smoothness of the pipe connecting them. The objective function Lai and Schaake used was

$$\sum_{(i,j)} a\, L_{ij}\, D_{ij}^{e} + b\left[\sum_{(i,j)} Q_{ij}\, \Delta H_{ij} + \sum_{j} C_j H_j\right] \tag{12}$$

a and b are constants which account for unit conversion, a present value factor, etc. e is a coefficient whose value is determined through analysis of pipeline cost data. The first sum is the capital cost of the pipelines, the second is the cost of energy lost in flow through the pipes, and the third is the cost of the residual energy at the supply nodes. There are no pumps within the network, and the energy terms represent the cost of supplying water to the network from the external source by a pump. By making the substitution $Y_{ij} = D_{ij}^{2.63}$ the constraints (11) become linear. Using eq. (1) for the flows, the objective function (12) becomes:

$$\sum_{(i,j)} (a\, L_{ij}\, Y_{ij}^{e/2.63} + b_{ij}\, Y_{ij}) \tag{13}$$

where the coefficients b_{ij} are based on the heads and flows. Minimization of (13) subject to constraints of type (11) for all supply nodes and to non-negativity of Y_{ij} is a linear program (which Lai and Schaake solved by a self-developed iterative LP program). The method was used in a study of New York City's primary distribution system[10] in which several performance criteria were also introduced as

objectives (e.g. sum of the residual pressures at all the supply nodes, the residual pressure at the farthest supply point, etc.).

Kally[23] approached the problem by extending the LP formulation for branching systems. His reasoning is as follows. In a branching system, if one changes the length of a pipe segment (which has some fixed diameter) the resulting changes in the heads at nodes are linearly proportional to the magnitude of this change. In a looped network the same effect is non-linear, due to the redistribution of flows once the design is changed. Still, if the change in length of a segment is small enough the resulting change in heads is approximately linearly proportional to the change in length. The ultimate decision variables in Kally's formulation are the lengths of the segments, the same as was for a branching network, and the objective function is also the same. The problem is solved through a sequence of linear programmes; the decision variables in each LP are the lengths in each link along which the diameter is to be changed from one diameter to another, i.e. the length to be taken away from one segment and given to another. Denoting by S_{ijm} the length in link (i,j) of change from the present diameter to diameter m (of a candidate list), these changes have to satisfy

$$+\sum_{m} S_{ijm} \leqslant L_{ij} \tag{14}$$

that is, the sum of all changes cannot exceed the total length of the link. Also, considering the minimum heads required at certain nodes, $HMIN_j$, the changes in diameters are limited by

$$\sum_{(i,j)} \sum_{m} \left(\frac{\Delta H_k}{\Delta s_{ijm}}\right) s_{ijm} \leqslant (H_k - HMIN_k) \qquad (15)$$

where H_k is the head at node k in the present iteration, and $(H_k - HMIN_k)$ is therefore the extra head which can still be eliminated (if feasible from other considerations). The coefficients $(\Delta H_k / \Delta s_{ijm})$ are the linear approximations for the rate of change of head with respect to changes in segment lengths. Kally's approach was to obtain these values as the difference in heads at the nodes between two solutions - one with existing segments, and another with one $\Delta s_{ijm} = 1$. For each $\Delta s_{ijm} = 1$ one has to run a network solver and obtain a flow solution, then one computes the head differences Δh_k between it and the "basic" flow solution - the one with all $\Delta s_{ijm} = 0$.

The objective function for each iteration's LP is

$$\text{Min} \sum_{i,j} \sum_{m} C_{ijm} \, s_{ijm} \qquad (16)$$

where C_{ijm} is the cost of changing one unit length of pipe in link (i,j) from its present diameter to diameter m.

Upon solution of the LP, the changes of diameters over lengths s_{ijm} are introduced and a new iteration is begun: a "basic" flow solution is computed, a series of flow

solutions for all $\Delta s_{ijm} = 1$ are obtained and (16) is minimized, subject to (14), (15) and non-negativity of the s.

Kohlhass and Mattern[25] used linear programming in optimizing a looped network in which the heads are fixed in advance - a condition similar to the one stipulated by Lai and Schaake[26].

Recently, a more general method has been developed[2] and introduced into practice. The method called Linear Programming Gradient method (LPG) is based on the following reasoning. If the flows throughout a looped network are known then its optimal design can be obtained by an LP formulation similar to that for a branching network. The optimization has, therefore, to find the optimal flow distribution, and for it the optimal design. This is achieved by a hierarchical approach. In the lower level of the hierarchy the optimal design for a particular flow distribution is obtained by LP; in the higher level, the flow distribution is modified, using certain results of the LP solution, towards an optimal flow distribution. This procedure is continued iteratively until some termination criterion is met.

The LP for a fixed flow distribution in a looped network operating under gravity for a single loading is: minimize (8), subject to (5) and (7), to non-negativity of the X, and to the additional constraint:

$$\sum_{(i,j)\in p} \sum_{m} J_{ijm} X_{ijm} = b_p \tag{17}$$

where p designates a path in the network, and b_p is the (known) head difference between its ends (all other notation is the same as in Section 4.1). Eq. (17) has to hold for all loops in the network with $b_p = 0$. When pumps, valves or reservoirs are to be included, the objective function and constraints have to be augmented, as will be explained below.

Having added constraints (17), the LP can be solved, and the set of optimal segments will be such that the network is hydraulically balanced by virtue of the fact that the constraints (17) have been satisfied. Denoting by $\vec{Q}$ the vector of flows in all the paths, which may be any arbitrary set of flows as long as they satisfy continuity at all nodes, then the optimal cost of the network, F, for this $\vec{Q}$ can be written as

$$F = LP(\vec{Q}) \tag{18}$$

where LP denotes that F is the outcome of a linear program. Next $\vec{Q}$ is modified in a way which approaches optimality. Denoting by $\overrightarrow{\Delta Q}_p$ the change of flow in path p, then

$$\frac{\partial F}{\partial(\Delta Q_p)} = \frac{\partial F}{\partial b_p}\cdot\frac{\partial b_p}{\partial(\Delta Q_p)} + \sum_{r\in R}\frac{\partial F}{\partial b_r}\cdot\frac{\partial b_r}{\partial(\Delta Q_p)}$$

$$= W_p\frac{\partial b_p}{\partial(\Delta Q_p)} + \sum_{r\in R} W_r\frac{\partial b_r}{\partial(\Delta Q_p)} \tag{19}$$

where W_p and W_r are the dual variables of constraint (17) for the paths p and r, respectively, where R are the paths which share a link (or more than one link) with path p. Using Eq. (17) and the definition of J from eq. (1):

$$\frac{\partial b_p}{\partial(\Delta Q_p)} = \frac{\partial b_p}{\partial(Q_p)} = \sum_{(i,j)\in p} \sum_m 1.852 \; Q_{ij}^{0.852} \; C_{ijm}^{-1.852} \cdot D_{ijm}^{-4.87} \; X_{ijm}$$

$$= 1.852 \sum_{(i,j)\in p} \frac{1}{Q_{ij}} \sum_m \Delta H_{ijm} \qquad (20)$$

$\partial(\Delta Q_p) = \partial(Q_p)$ because both are incremental changes in the flow in the path. Q_{ij} and ΔH_{ijm} have already been used in setting up the LP, so once it has been solved and the duals W_p and W_r are known, the components of the gradient.

$$G_p = \frac{\partial F}{\partial(\Delta Q_p)} = 1.852 \; W_p \sum_{(i,j)\in p} \frac{1}{Q_{ij}} \sum_m \Delta H_{ijm} \pm \sum_{r\in R} W_r \sum_{(i,j)\in r} \frac{1}{Q_{ij}} \sum_m \Delta H_{ijm} \qquad (21)$$

can easily be computed. The sign of the additional terms is positive when path r used link (i,j) in the same direction as path p, and negative otherwise. With these components one can define a vector change in path flow, $\overrightarrow{\Delta Q}$ such that

$$LP(\vec{Q} + \beta \overrightarrow{\Delta Q}) \leq LP(\vec{Q}) \qquad (22)$$

is a step size, which is selected by an appropriate one-dimensional search procedure.

Several loadings should be considered in the design. Maximum hourly flows during the day and fire fighting demands are normally used, but often the low demands, as may occur at night, have to be considered as well. For each loading, an initial flow distribution has to be specified

which satisfies continuity at all nodes. A number of head constraints (7) are written for each loading, and the entire set is included in a single LP matrix. Once the LP has been solved, the gradient section modifies the flow distribution for each of the loadings, using the results of the LP.

Since the initial flows for each loading are quite arbitrary, there may not exist a set of segment diameters such that the head line constraints(17) are satisfied for all loadings. Therefore, two new variables are added in each of these constraints - one with a positive and the other with a negative sign, and both required to be non-negative. These variables are assigned a large penalty coefficient in the objective function and can therefore be viewed as artificial variables. Each such variable may be viewed as a dummy valve in the path, able to take up the excess head for the specified flow in the path. If a dummy valve does appear in the final optimal solution, this means that a real valve will have to be installed at the location specified, and operated accordingly. When an actual valve does exist in the system it is represented in the LP by the head loss it provides under each loading.

The iterative LPG procedure is terminated when any of several criteria is met (no significant improvement from one iteration to the next, specified number of flow change iterations exceeded, etc.).

When a pump is to be designed, the head it has to add for each of the loadings is the decision variable. These variables are introduced into the constraints (7) and (17) with the proper sign. An iterative procedure is used in dealing with the non-linear cost vs. head function for pumps. For reservoirs, the decision variable is the elevation at which it is to be located. This elevation appears in all constraints for paths ending at the reservoir. A linear cost vs. elevation relation is used in the objective function.

2.5.2 Non-linear Programming Models

An early effort[34] was made to use a gradient-like technique in optimizing the design of a pipe network under one loading. The original work, which was not published, constituted the basis for Lemieux's thesis[27]. Pipe diameters are changed, one pipe at a time, according to the derivative of objective function with respect to pipe diameters. After each change in a diameter the new network is solved, using the Newton-Raphson method, and the last Jacobian of this solution appears in the computation of the derivatives.

Pitchai[29] formulated a non-linear integer programming problem in seeking the optimal diameters of a network, and solved it by combining random search and examination of adjacent design points. Jacoby[20] used a gradient-approximation method to seek minimum of a merit function which combined the objective function and penalties for violating head and continuity constraints for a single loading. Since

the search is conducted in the region which is hydraulically infeasible (it is an exterior point method), if the search terminates prematurely one may not have a feasible hydraulic design. A more detailed analysis of Jacoby's paper may be found in Reference 35.

Cembrowicz and Harrington[4] have dealt with minimization of the capital cost of a pipe network designed to operate under one loading. Using graph theory they claim to decompose the problem such that the non-convex objective function is broken into subsets of convex functions. Each function relates to a pipe or loop and is minimized separately, using a method of feasible directions. The number of optimizations may be very large and they have to be scanned to locate the global optimum, so that computationally the method does not seem practical.

Watanatada[40] has developed a method for optimal design of pipeline networks supplied at a number of nodes, and applied it to real networks of moderate size. For a network with P pipes and M nodes, of which MS are supply nodes, the problem is

$$\text{Min} \quad C_T(D, H, Q) = \sum_{p=1}^{P} U_p L_p + \sum_{k=1}^{MS} S_k \,, \tag{23}$$

subject to

$$QR_k(D, H, Q) = 0, \qquad k = 1,\ldots,M, \tag{24}$$

$$D_p \geqslant DMIN, \qquad p = 1,\ldots,P \tag{25}$$

$$H_k \geqslant HMIN_k, \qquad k = 1,\ldots,M \tag{26}$$

$$-\ QN_k \geqslant 0, \qquad k = 1,\ldots,MS \tag{27}$$

where D = a vector of P pipe diameters,

H = a vector of M node heads,

Q = a vector of MS supply rates at supply nodes,

$U_p = U_p(D_p)$, the cost per unit length of pipe as a function of its diameter,

$S_k = S_k(QN_k,\ H_k)$, the cost of supplying QN_k at a head H_k,

QR_k = algebraic sum of flows leaving the node.

This constrained optimization formulation was converted into an unconstrained one by using a variable transformation due to Box[3]. Utility variables Z_i, $i = 1,\ldots,(P + M + MS)$ are defined by

$$D_p = DMIN + Z_p^2, \qquad p = 1,\ldots,P \tag{28}$$

$$H_k = HMIN_k + Z_{p+k}^2, \qquad k = 1,\ldots,M \tag{29}$$

$$QN_k = -\ Z_{P+M+k}^2, \qquad k = 1,\ldots,MS \tag{30}$$

and the problem now becomes

$$\text{Min} \quad C_T(Z) \tag{31}$$

subject to $QR_k(Z) = 0, \qquad k = 1,\ldots,M$ (32)

A new function is now defined by combining constraints (32) with the objective function (31) according to the method suggested by Haarhoff and Buys[18]:

$$\text{Min}\left[F^r(Z) = C^T(Z) + \sum_{k=1}^{M} E_k^r QR_k(Z) + W \sum_{k=1}^{M} QR_k^2(Z)\right] \tag{33}$$

The subscript r is an iteration counter; E^r and W are penalty multipliers - the first is updated at each iteration and the second is a preassigned fixed constant. The variable metric method of Fletcher and Powell[14] was used to minimize (33), and was found to be superior to the Fletcher and Reeves method of conjugate directions[15], even though the former required more computer memory. Watanatada's method has the advantage that the flow solution is incorporated directly into the optimization, and one therefore does not need a network solver as a separate program. The danger is, however, that if the procedure terminates prematurely the solution may not be feasible hydrolically.

A method based on Abadie's[1] GRG method was developed by Shamir[36], in which the design and operation under a number of loadings are to be optimized. For a network with N nodes, operating under L loadings the problem is:

$$\text{Min} \quad F(d, u, x, s) = f(d) + \sum_{l=1}^{L} w^l c^l(d, u^l, x^l, s^l) , \tag{34}$$

$$\text{subject to} \quad d \in D \tag{35}$$

$$u^l \in U^l , \quad \forall\, l \tag{36}$$

$$G^l(d, u^l, x^l, s^l) = 0 \;\forall\, l \tag{37}$$

$$x^l = \left[x / G^l(d, u^l, x^l, s^l)\right] = 0 \in X^l \;\forall\, l \tag{38}$$

where d = the design variables (pipe diameters, pump capacities), which have to belong to the set D.

u^l = the operation variables (valves and pumps ON/OFF) for the l^{th} loading, which have to belong to the set U^l.

x^l = the dependent variables (heads, consumptions) of the l^{th} flow solution, which have to be within ranges given by X^l.

s^l = the independent (fixed) variable of the l^{th} flow solution.

f = cost function of the design.

C^l = cost of operation for the l^{th} loading.

w^l = weights.

$[G^l] = 0$ = a set of simultaneous node continuity equations (eq. (1)) for the l^{th} loading. The individual equations are $G^l_j = 0$, $j = 1, \ldots, N$.

It is assumed that the sets D, U^l and X^l simply specify a range of values for the corresponding variables. The L sets of equations (37) are combined with the objective function (34) to form the Lagrangian:

$$\mathcal{L}(d, u, x, s, \lambda) = F(d, u, x, s) + \sum_{l=1}^{L} \sum_{j=1}^{N} {}^l_j G^l_j(d, u^l, x^l, s^l)$$

$$= F + [G]^T \cdot [\lambda] \qquad (39)$$

where T stands for the transpose. At any point (d, u) the following has to hold

$$\left[\frac{\partial \mathcal{L}}{\partial x}\right] = 0 = \left[\frac{\partial F}{\partial x}\right] + \left[\frac{\partial G}{\partial x}\right]^T \cdot [\lambda] \qquad (40)$$

Since G and x can be separated into independent flow problems, eqn. (40) can be decomposed into

$$\left[\frac{\partial \mathcal{L}}{\partial x^l}\right] = 0 = \left[\frac{\partial F}{\partial x^l}\right] + \left[\frac{\partial G^l}{\partial x^l}\right]^T \left[\lambda^l\right] ,$$

$$l = 1,\ldots,L \qquad (41)$$

The Langrange multipliers are therefore solved in L groups of N values each from eqn. (41). The matrix $\left[\partial G^l/\partial x^l\right]$ is the last Jacobian of the l^{th} flow solution by the Newton-Raphson method, and is therefore available directly as a by-product of the flow solution. Once the 's have been obtained, the components of the (reduced) gradient are computed from

$$\left[\nabla F\right] = \begin{bmatrix} \nabla d \\ \\ \nabla u \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial d} \\ \\ \frac{\partial \mathcal{L}}{\partial u} \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial d} \\ \\ \frac{\partial F}{\partial u} \end{bmatrix} + \left[\frac{\partial G}{\partial d}, \frac{\partial G}{\partial u}\right]^T \left[\lambda\right] \qquad (42)$$

At a (local) optimum $\nabla F = 0$. At any other point $-\nabla F$ points in the direction of the steepest descent of F, while changes in flow solutions due to a move in this direction are already taken into account. A move in this direction is now made, using a one-dimensional search procedure. At the new point the L flow problems are solved and the reduced gradient calculation repeated. The constraints on d, u^l, and x^l are used in selecting the step size in each move. The search is terminated by given criteria for improvement between iterations, value of the gradient, number of iterations etc.

Because flow solutions are computed at each step, and because the constraints on the decision variables are not violated during the moves, the current solution is feasible (this is essentially an interior point method), and if it terminates prematurely, one at least has a feasible solution - which is better than the one having started with.

2.5.3 Other Methods

Deb and Sarkar[7] based their method on the concept of equivalent diameters, for least cost design of a network operating under a single loading and in which the heads were assumed to be known. The results are therefore quite limited in application.

Deb[6] again used a similar combination of the flow and cost equation, and developed a method for optimal design of a system consisting of a pumping station, an elevated reservoir and a pipe network fed from it by gravity. The results are further restricted by the fact that the shape of pressure surface over the network is assumed to have a specific form.

2.6 Operation Over Time

Some of the methods described above can be used to reach optimal operating decisions for existing networks. This is done by fixing all design variables at their actual values and carrying out the optimization for the operational variables. Special methods have, therefore, to be developed

for optimization of operation over time, in which the only decision variables are the operation of pumps and the setting of valves during the specified time horizon.

Dreizin et al.[11] used a hydraulic simulator of a particular water system as the basic building block in a program which attempted to improve operating policies. The decision variables were those water levels (called set-points) in specified reservoirs at which pumps are to be switched ON or OFF. No algorithmic optimization was found for solving the problem, and a sequence of simulations with response surface analysis (a gradient-like search) was used.

Some work in the City of Philadelphia[8] resulted in selection of operating policies over a day based on a comparison of costs for several proposed policies. No optimization was attempted.

Sterling and Coulbeck[38,39] optimized pumping costs in a water system using dynamic programming and a two level hierarchical approach. A group at the University of Cambridge, England, has been engaged in development of on-line control and optimization of the operation of regional water systems[12,13] which are being implemented and tested in the field.

3. MODEL FORMULATION

The model presented here is developed by Alperovits and Shamir[2]. The model uses Linear Programming Gradient (LPG) method to optimize the solution and is the first to incorporate flow solution into the optimization procedure, without making any assumptions about the nature of pressure surface as attempted by the earlier investigators.

The Linear Programming Gradient method deals with looped networks and decomposes the optimization problem into a hierarchy of 2 levels as depicted in Figure 1.

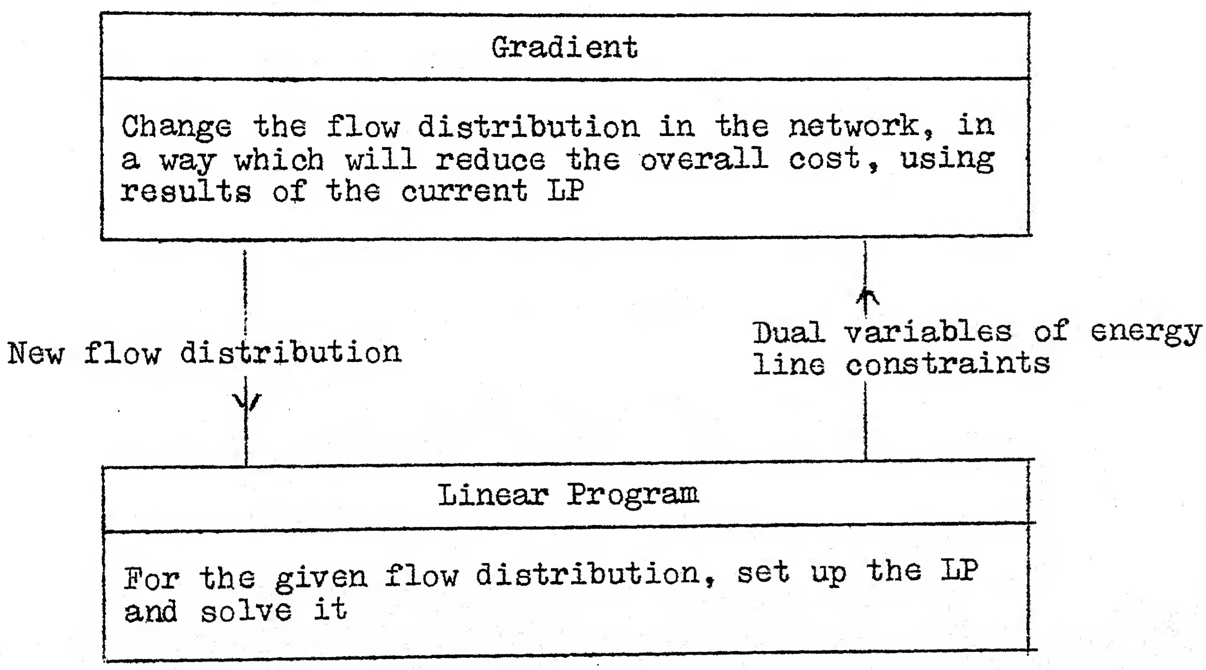


Figure 1. Schematic Diagram of the LPG Method.

The first step in developing the LPG method is to consider optimization of the design when the distribution of flows in the network is assumed to be known. We adopt the formulation given by objective function (8) and constraints(5 (7),(17)in which the lengths of the segments of candidate diameter in each link are the decision variables. For any Q the optimal cost of the network may be written as

$$\text{Cost} = \text{LP}(\vec{Q})$$

The next stage is to develop a method for systematically changing Q with the aim of improving cost. The method for changing Q is based on the use of the dual variables, which aid in defining a gradient move. As the flow distribution should always be balanced at the nodes hence equal corrections (sign of the correction depending on the flow direction) are applied to each pipe in a given loop. Shamir[2] proposed that the component of the gradient vector G corresponding to loop p should be as

$$G_p = \frac{\partial(\text{Cost})}{\partial(\Delta Q_p)} = W_p \sum_{i,j} \left(\frac{1}{Q_{ij}}\right) \sum_m \Delta H_{ijm} \qquad (43)$$

Summations are performed only for the appropriate links, i.e., those belonging to the p^{th} loop.

Quindry et.al.[30] modified the gradient vector G taking into consideration the interactions of paths with each other as well and proposed their formulation for G_p as given in equation (19). Their formulation was found to give

better results compared to equation (43). The proof for the equation (19) is given in Appendix 1.

3.1 Incorporating Pumps in the Formulation

When there are to be pumps in the system, the locations at which pumps may be installed are selected by the designer, but since program can set certain pump capacities to zero, if that is the optimal solution, the program actually selects the locations at which pumps will be installed. The decision variable associated with each location is the head to be added by the pump. If one denotes by XP(t) the head added by pump number t, then the head constraints of the type (7) for paths with pumps become

$$HMIN \leqslant H_s + \sum_t XP(t) \pm \sum_{i,j} \sum_m J_{i,jm} x_{i,jm} \leqslant HMAX \qquad (44)$$

where the first summation is over the pumps in the path. For any path which has pumps in it, be it a closed loop or an open path, equation (17) had to be modified in a similar manner, and it then becomes

$$\sum_{i,j} \sum_m J_{ijm} x_{ijm} \pm \sum_t XP(t) = b_p \qquad (45)$$

In equations (44) and (45) the signs in front of the various terms depend on the direction of flow.

The decision variables for the pumps XP(t) have to be introduced linearly into the objective function if the problem is to remain a linear program. This is done by

considering the cost of the pump as a linear functions of the capacity, i.e., its rated horsepower. However, the cost per horse-power decreases with increasing pump capacity. Successive approximations are used in the program to cope with the non-linearity of the cost curve. The power needed to operate the pump is given by

$$HP = Y \cdot Q \cdot XP/\eta \tag{46}$$

where Y is a coefficient, Q is the flow, XP is the head added by the pump, and η is the efficiency. For a fixed discharge and a fixed efficiency, equation (46) becomes

$$HP = K \cdot XP \tag{47}$$

where K is a constant.

The operating cost of the pump is therefore linearly proportional to the decision variable, which is XP. An iterative procedure is developed to take into account the non-linearity of the capital cost as follows:

(1) Assume values for the cost per HP for each pump location.

(2) Solve the linear program with these values as the coefficient of the XP in the objective function.

(3) For the resulting XP after the LP has been solved, compute the cost per HP. If all values are close to those assumed, this step is complete and one proceeds to a flow iteration by the gradient method. Otherwise one takes the new costs and solves the program again.

This procedure is found to work well.

3.2 Incorporating Reservoirs in the Formulation

The decision variable for a reservoir is the elevation at which it is to be located. An initial elevation is assumed, then XR is the additional elevation where the reservoir is to be located, relative to its initially assumed elevation (XR^+ is addition to reservoir height and XR^- is deduction from its height). Path equations have to be formed between the reservoir at node s and nodes in the network. For node n

$$HMIN_n \leqslant HO_s + XR_s^+ - XR_s^- \pm \sum_{i,j} \sum_m J_{ijm} \, x_{ijm} \leqslant HMAX_n \tag{48}$$

where HO_s is the initial elevation of the reservoir at node s, and XR_s is the additional elevation to be selected by the program. The coefficient of XR_s in the objective function is the cost of altering the location of the reservoir by 1 unit (1 m).

3.3 Selection of Candidate Diameters

At the outset, the list of candidate diameters for each link is based on a suitable hydraulic gradient say 0.001. Out of the available pipe diameters, the diameter is selected which gives the nearest hydraulic gradient to the above value. Candidate list consists of 3 pipe diameters, one larger and one smaller than the one selected above.

If the first selection of candidate diameters results in an optimal LP solution, the list may have to be modified after the flows in the network have been changed by the gradient move. The modifications in the list of the candidate diameters are based on the following rules:

(a) If in the optimal LP solution a link is made entirely of one diameter; then for the next LP, the list is made of 3 diameters, the existing one and both its neighbours.

(b) When in the optimal solution a link is made of 2 diameters, the list for the next LP is made of these two, plus one adjacent to that diameter which has the larger of the two lengths.

Both cases result in a list of only 3 diameters for each link. In going from one LP solution to the other, the list for a particular element may remain unchanged, or a diameter may be dropped off one end of the list and a new one added at the other.

3.4 Overview of the Algorithm

(a) The first step is to decide about the initial flow distribution knowing the demand at each of the nodes.

(b) Objective function and constraints are formulated as described previously.

(c) The LP is solved to give the optimum results for the initial flow distribution. The gradient vector is calculated knowing the dual variables of the LP.

(d) The magnitude of the gradient vector is calculated as follows

$$\text{MAGNITUDE} = (G_1^2 + G_2^2 + \dots + G_n^2)^{1/2} \tag{49}$$

where $G_1, G_2, \dots, G_n$ are components of gradient vector. If the MAGNITUDE is less than a prespecified value the problem is solved. If not so, the flows are to be changed in the gradient direction.

(e) The gradient vector is normalized by dividing every component by MAGNITUDE.

(f) The gradient vector is multiplied by a trial step length to give the required flow corrections in loops.

(g) After applying these corrections, if any of the flow direction changes, the step length is reduced by a factor (say half) and new flow corrections are calculated. Program is terminated if the step length becomes less than a prespecified value.

(h) For the new flow pattern, the LP is formulated and solved.

(i) If the optimum cost of the new LP comes out to be less than the preceding LP, the new gradient vector is calculated and control is shifted to step (d).

If the optimum of the new LP is more than the preceding one then the step length is reduced by a factor and the preceding gradient move and flow pattern is used to get the new flow pattern and control is shifted to step (h). If the step length becomes less than a prespecified value, the program is terminated and the present solution is taken to be the optimum.

4. DESIGN OF WATER DISTRIBUTION SYSTEM USING THE PROGRAM

A computer program in Fortran has been written based on the LPG method as given in the last section. The program can be used to design a new water distribution system given the consumption pattern during peak hours at various nodes of the system. Some salient features regarding the use of the program are as given below:

i) The critical consumption pattern, i.e. the peak demand should be known.

ii) An initial flow distribution along the pipes is to be given as input data. The given flow distribution should satisfy the continuity of flow requirement at all nodes of the system.

iii) In case of more than one source of supply, the amount of water to be withdrawn from different sources should be decided beforehand, i.e. the program does not take into consideration the optimal allocation of resources.

iv) The suitable positions for the booster pumps should be given as input data. If a pump is required at a specified position to minimize the cost, the program will give the output in terms of the head to be added by the pump at that point.

Example Problems

The program is tested for a set of problems with increasing network size. Execution time for different problems is reported for Dec-10 system (available at IIT/Kanpur). Details of problems along with solutions are as follows:

The set of available diameters and the cost of pipes and pumps are common to all problems. The cost of pumps is taken to be Rs. 1000/- per installed horse-power. The cost data for pipes are given in Table 1. The cost for the source of supply is to be given in terms of the increase in cost for 1 m increase in head at the supply node. An initial head is assumed at the supply node and the program gives the increase or decrease in head required to minimize the cost. As the lives of various components in the distribution system are different so all the capital costs are converted to annual costs and the total annual cost of the system is minimized.

Capital costs can be converted to annual costs by multiplying by the Capital Recovery Factor (CRF).

$$CRF = \frac{i(1 + i)^n}{(1 + i)^n - 1}$$

in which i is the rate of interest and n is the estimated life of the component. Assuming 10% rate of interest and 15 years and 40 years to be the lives of pumps and pipes respectively the factors come out to be:

$$\text{CRF (Pump)} = \frac{0.1(1 + 0.1)^{15}}{(1 + 0.1)^{15} - 1} = 0.13147$$

$$\text{CRF (Pipes)} = \frac{0.1(1.1)^{40}}{(1.1)^{40} - 1} = 0.10225$$

To calculate the increase in cost due to increase of supply head of pump by 1 m, the following procedure is adopted:

Assuming the pump cost is Rs. 1000/- per installed horse-power:

Hence the increase in cost due to 1 m increase in head

$$= 1000 \times 2.7323 \times 10^{-4} \times Q = 0.27323 \times Q \text{ Rs/m}$$

where Q is water supplied by pump in lpm.

Assuming that the pump operates at this rate for 8 hours a day, the increase in annual cost due to operation

$$= \frac{0.163 \times Q \times 8 \times 365 \times 0.25}{1000 \times 0.8} = 0.14873\ Q \text{ Rs/m}$$

where cost of power = Rs. 0.25/kwh and efficiency of pump = 0.8.

Hence increase in cost due to 1 m increase in supply head

$$= (\text{Capital cost} \times \text{CRF}) + \text{Operation cost}$$

$$= \left[(0.27323 \times 0.13147) + 0.14873\right] Q$$

$$= 0.18465 \times Q \text{ Rs/m}$$

Problems 1 to 8: Distribution networks for the problems are given in the following figures:

Problem 1 - Figure 2

Problem 2 - Figure 3

Problem 3 - Figure 4

Problem 4 - Figure 5

Problem 5 - Figure 6

Problem 6 - Figure 7

Problem 7 - Figure 8

Problem 8 - Figure 9

Every problem has a pump serving as the source and a booster pump. Node no. 1 is supply node in each problem and rest all nodes have 100 lpm demand. Hence supply at node 1 is given as follows:

Problem 1 - 600 lpm

Problem 2 - 800 lpm

Problem 3 - 900 lpm

Problem 4 - 1100 lpm

Problem 5 - 1200 lpm

Problem 6 - 1400 lpm

Problem 7 - 1500 lpm

Problem 8 - 1600 lpm

The minimum pressure allowed at each node is taken to be 15 m.

The location of booster pump to be provided in each problem is given as follows:

Problem 1 - Pipe No. 2

Problem 2 - Pipe No. 6

Problem 3 - Pipe No. 6

Problem 4 - Pipe No. 6

Problem 5 - Pipe No. 6

Problem 6 - Pipe No. 6

Problem 7 - Pipe No. 6

Problem 8 - Pipe No. 9

The initial flow distribution and candidate diameters for each pipe are given in the following tables:

Problem 1 - Table 2

Problem 2 - Table 3

Problem 3 - Table 4

Problem 4 - Table 5

Problem 5 - Table 6

Problem 6 - Table 7

Problem 7 - Table 8

Problem 8 - Table 9

The structures of initial linear programmes are given in following tables:

Problem 1 - Table 10

Problem 2 - Table 11

Problem 3 - Table 12

Problem 4 - Table 13

Problem 5 - Table 14

Problem 6 - Table 15

Problem 7 - Table 16

Problem 8 - Table 17

Increase in cost due to 1 m increase in supply head for various problems is as following:

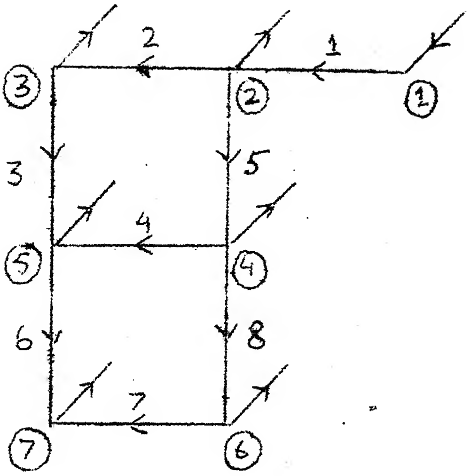
Problem 1 - 110.79 Rs/m

Problem 2 - 147.67 Rs/m

Problem 3 - 166.18 Rs/m

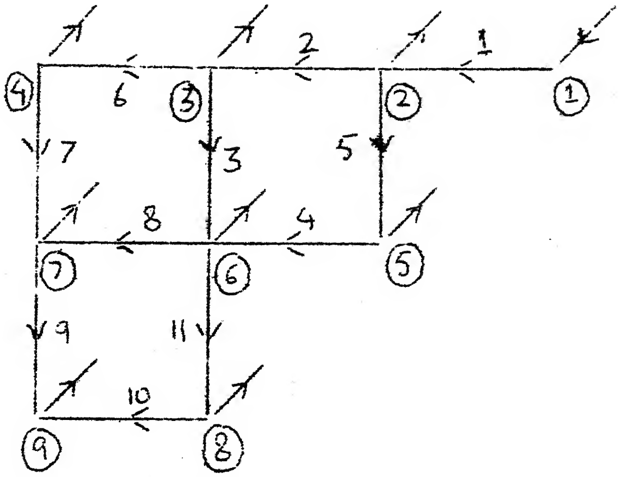
Problem 4 - 203.11 Rs/m

Problem 5	-	221.58 Rs/m
Problem 6	-	258.51 Rs/m
Problem 7	-	276.97 Rs/m
Problem 8	-	295.44 Rs/m



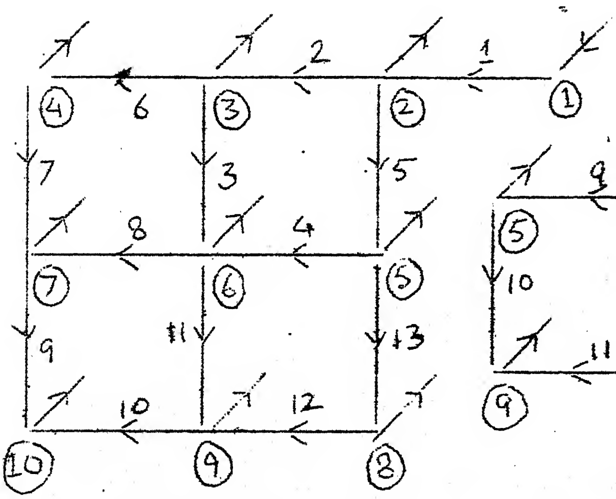
PROBLEM-1

FIG-2



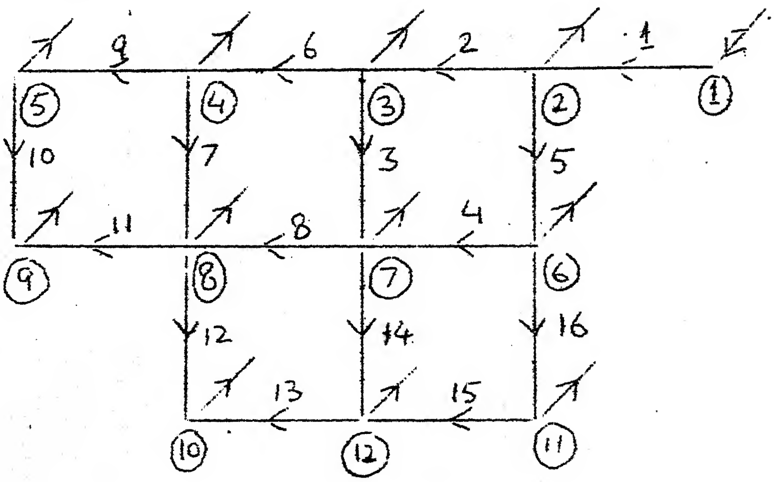
PROBLEM-2

FIG-3



PROBLEM-3

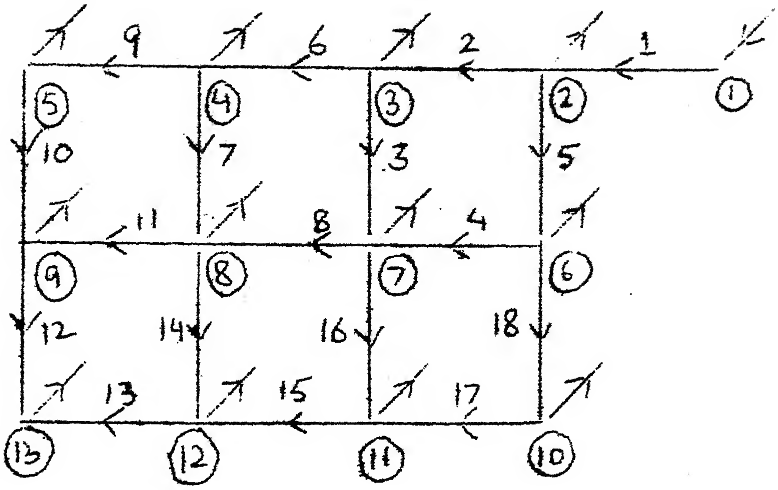
FIG-4



PROBLEM-4

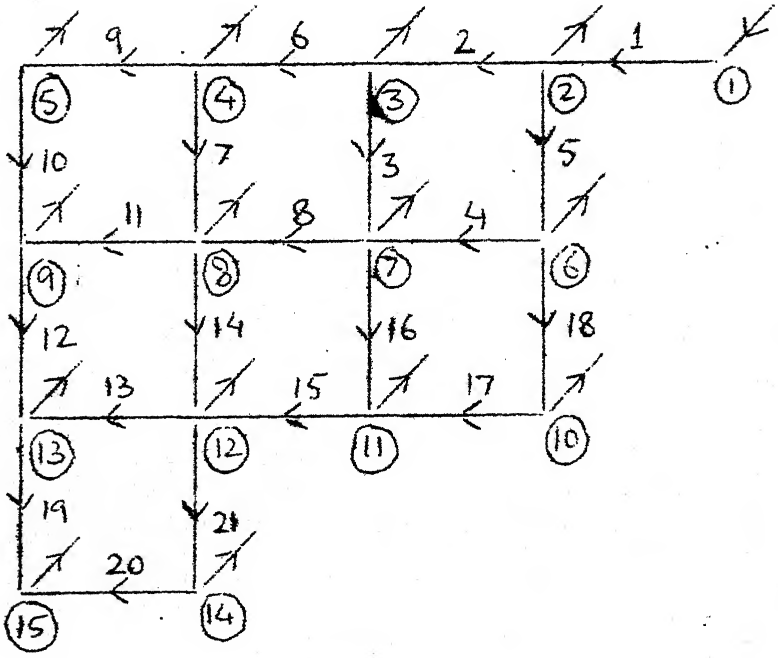
FIG-5

OTE: PIPE NUMBERS AND FLOW DIRECTIONS ARE SHOWN ALONG THE PIPES. NUMBERS WITHIN CIRCLES INDICATE NODES.



PROBLEM-5

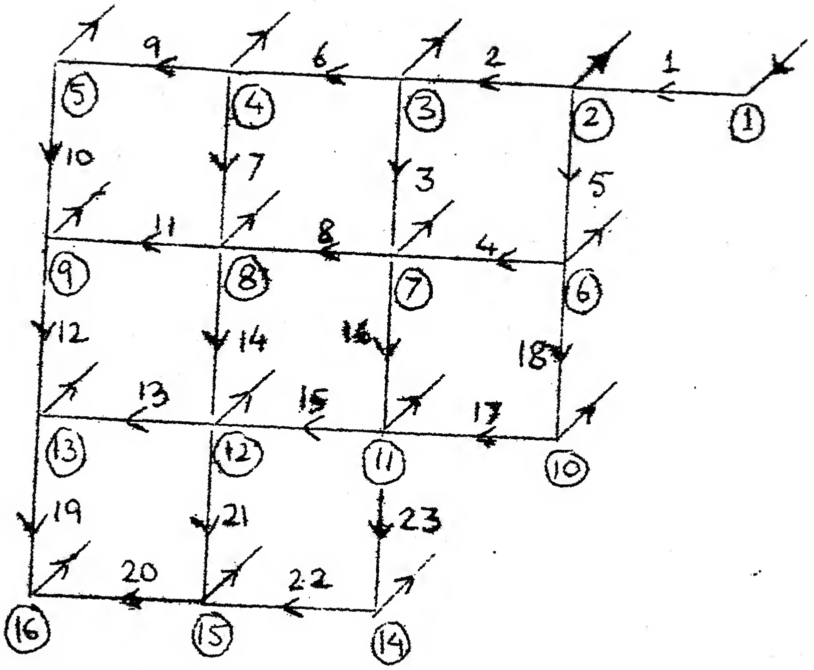
FIG-6



PROBLEM-6

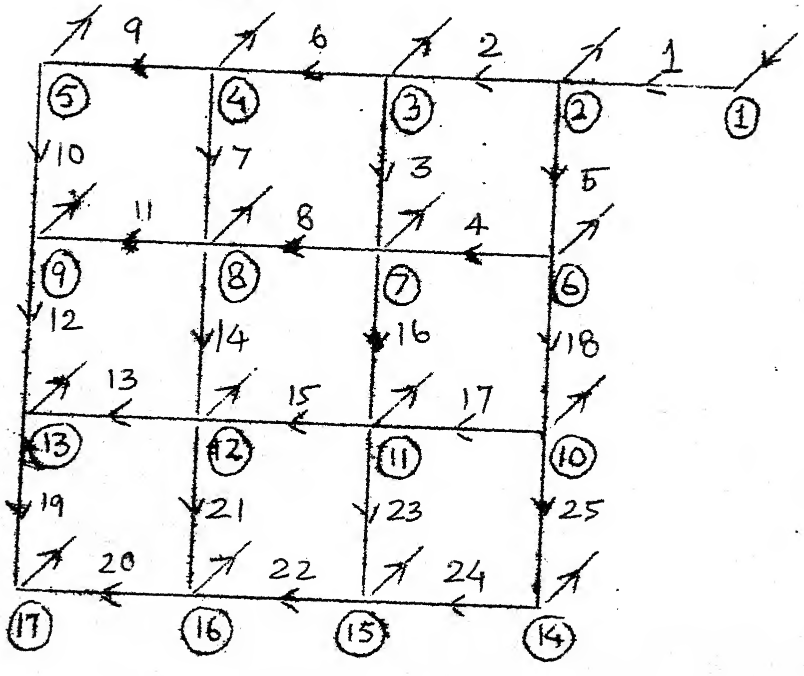
FIG-7

NOTE: PIPE NUMBERS AND FLOW DIRECTIONS ARE SHOWN ALONG THE PIPES, NUMBERS WITHIN CIRCLES INDICATE NODES.



PROBLEM-7

FIG-8



PROBLEM-8

FIG-9

Table 1

Available Pipes, Their Costs, and Hazen-William Coefficients

CRF for pipes = 0.10225

S.No.	Pipe diameter (mm)	Cost Rs/m	Annual cost Rs/m	Hazen-William coefficient
1	15.0	2.00	0.2045	140.0
2	20.0	3.00	0.30675	140.0
3	25.0	4.50	0.46012	140.0
4	32.0	5.00	0.51125	140.0
5	40.0	7.00	0.71575	140.0
6	50.0	8.00	0.818	140.0
7	63.0	11.00	1.12475	140.0
8	80.0	15.50	1.58487	140.0
9	100.0	22.00	2.2495	140.0
10	125.0	35.50	3.62987	140.0
11	150.0	47.00	4.80575	140.0
12	200.0	83.00	8.48675	130.0
13	250.0	112.00	11.452	130.0
14	300.0	145.00	14.82625	130.0
15	350.0	187.00	19.12075	130.0
16	400.0	221.00	22.59725	130.0
17	450.0	278.00	28.4255	130.0
18	500.0	329.00	33.64025	130.0
19	600.0	463.00	44.34175	130.0

Note:

1) According to the present practice in U.P. Jal Nigam, upto 150 mm diameter, PVC pipes are provided and beyond this size, AC pipes are used. The costs and Hazen-William coefficients are taken accordingly.

2) 15 mm pipe has also been taken in the set of available pipes in the present case but usually such small pipes should not be considered while designing the distribution system. The optimum results will come out to be different if small pipes are not included in the list of available pipes.

Table 2

Pipe Data for Problem - 1

Pipe No.	Length (m)	Initial Flow (lpm)	Candidate diameters (mm)
1	1000.0	600.0	150, 200, 250
2	1000.0	280.0	100, 125, 150
3	1000.0	180.0	80, 100, 125
4	1000.0	10.0	32, 40, 50
5	1000.0	220.0	100, 125, 150
6	1000.0	90.0	63, 80, 100
7	1000.0	10.0	32, 40, 50
8	1000.0	110.0	80, 100, 125

Table 3

Pipe Data for Problem - 2

Pipe No.	Length (m)	Initial Flow (lpm)	Candidate diameters (mm)
1	1000.0	800.0	150, 200, 250
2	1000.0	590.0	150, 200, 250
3	1000.0	210.0	100, 125, 150
4	1000.0	10.0	32, 40, 50
5	1000.0	110.0	80, 100, 125
6	1000.0	280.0	100, 125, 150
7	1000.0	180.0	80, 100, 125
8	1000.0	10.0	32, 40, 50
9	1000.0	90.0	63, 80, 100
10	1000.0	10.0	32, 40, 50
11	1000.0	110.0	80, 100, 125

Table 4

Pipe Data for Problem - 3

Pipe No.	Length (m)	Initial flow (lpm)	Candidate diameters (mm)
1	1000.0	900.0	150, 200, 250
2	1000.0	580.0	150, 200, 250
3	1000.0	200.0	100, 125, 150
4	1000.0	10.0	32, 40, 50
5	1000.0	220.0	100, 125, 150
6	1000.0	280.0	100, 125, 150
7	1000.0	180.0	100, 125, 150
8	1000.0	10.0	32, 40, 50
9	1000.0	90.0	63, 80, 100
10	1000.0	10.0	32, 40, 50
11	1000.0	100.0	80, 100, 125
12	1000.0	10.0	32, 40, 50
13	1000.0	110.0	80, 100, 125

Table 5

Pipe Data for Problem - 4

Pipe No.	Length (m)	Initial flow (lpm)	Candidate diameters (mm)
1	1000.0	1100.0	200, 250, 300
2	1000.0	780.0	150, 200, 250
3	1000.0	200.0	100, 125, 150
4	1000.0	10.0	32, 40, 50
5	1000.0	220.0	100, 125, 150
6	1000.0	480.0	125, 150, 200
7	1000.0	190.0	100, 125, 150
8	1000.0	10.0	32, 40, 50
9	1000.0	190.0	100, 125, 150
10	1000.0	90.0	63, 80, 100
11	1000.0	10.0	32, 40, 50
12	1000.0	90.0	63, 80, 100
13	1000.0	10.0	32, 40, 50
14	1000.0	100.0	63, 80, 100
15	1000.0	10.0	32, 40, 50
16	1000.0	110.0	80, 100, 125

Table 6

Pipe Data for Problem - 5

Pipe No.	Length (m)	Initial flow (lpm)	Candidate diameters (mm)
1	1000.0	1200.0	200, 250, 300
2	1000.0	880.0	150, 200, 250
3	1000.0	200.0	100, 125, 150
4	1000.0	10.0	32, 40, 50
5	1000.0	220.0	100, 125, 150
6	1000.0	580.0	150, 200, 250
7	1000.0	200.0	100, 125, 150
8	1000.0	10.0	32, 40, 50
9	1000.0	280.0	100, 125, 150
10	1000.0	180.0	80, 100, 125
11	1000.0	10.0	32, 40, 50
12	1000.0	90.0	63, 80, 100
13	1000.0	10.0	32, 40, 50
14	1000.0	100.0	63, 80, 100
15	1000.0	10.0	32, 40, 50
16	1000.0	100.0	63, 80, 100
17	1000.0	10.0	32, 40, 50
18	1000.0	110.0	80, 100, 125

Table 7

Pipe Data for Problem - 6

Pipe No.	Length (m)	Initial flow (lpm)	Candidate diameters (mm)
1	1000.0	1400.0	200, 250, 300
2	1000.0	1080.0	150, 200, 250
3	1000.0	200.0	100, 125, 150
4	1000.0	10.0	32, 40, 50
5	1000.0	220.0	100, 125, 150
6	1000.0	780.0	150, 200, 250
7	1000.0	310.0	125, 150, 200
8	1000.0	10.0	32, 40, 50
9	1000.0	370.0	125, 150, 200
10	1000.0	270.0	100, 125, 150
11	1000.0	10.0	32, 40, 50
12	1000.0	180.0	80, 100, 125
13	1000.0	10.0	32, 40, 50
14	1000.0	210.0	100, 125, 150
15	1000.0	10.0	32, 40, 50
16	1000.0	100.0	80, 100, 125
17	1000.0	10.0	32, 40, 50
18	1000.0	110.0	80, 100, 125
19	1000.0	90.0	63, 80, 100
20	1000.0	10.0	32, 40, 50
21	1000.0	110.0	80, 100, 125

Table 8

Pipe Data for Problem - 7

Pipe No.	Length (m)	Initial flow (lpm)	Candidate diameters (mm)
1	1000.0	1500.0	200, 250, 300
2	1000.0	1180.0	200, 250, 300
3	1000.0	310.0	100, 125, 150
4	1000.0	10.0	32, 40, 50
5	1000.0	220.0	100, 125, 150
6	1000.0	770.0	150, 200, 250
7	1000.0	300.0	100, 125, 150
8	1000.0	10.0	32, 40, 50
9	1000.0	370.0	125, 150, 200
10	1000.0	270.0	100, 125, 150
11	1000.0	10.0	32, 40, 50
12	1000.0	180.0	80, 100, 125
13	1000.0	10.0	32, 40, 50
14	1000.0	200.0	100, 125, 150
15	1000.0	10.0	32, 40, 50
16	1000.0	210.0	100, 125, 150
17	1000.0	10.0	32, 40, 50
18	1000.0	110.0	80, 100, 125
19	1000.0	90.0	63, 80, 100
20	1000.0	10.0	32, 40, 50
21	1000.0	100.0	63, 80, 100
22	1000.0	10.0	32, 40, 50
23	1000.0	110.0	80, 100, 125

Table 9

Pipe Data for Problem - 8

Pipe No.	Length (m)	Initial flow (lpm)	Candidate diameters (mm)
1	1000.0	1600.0	200, 250, 300
2	1000.0	1170.0	200, 250, 300
3	1000.0	300.0	125, 150, 200
4	1000.0	10.0	32, 40, 50
5	1000.0	330.0	125, 150, 200
6	1000.0	770.0	150, 200, 250
7	1000.0	300.0	125, 150, 200
8	1000.0	10.0	32, 40, 50
9	1000.0	370.0	125, 150, 200
10	1000.0	270.0	100, 125, 150
11	1000.0	10.0	32, 40, 50
12	1000.0	180.0	80, 100, 125
13	1000.0	10.0	32, 40, 50
14	1000.0	200.0	100, 125, 150
15	1000.0	10.0	32, 40, 50
16	1000.0	200.0	100, 125, 150
17	1000.0	10.0	32, 40, 50
18	1000.0	220.0	100, 125, 150
19	1000.0	90.0	63, 80, 100
20	1000.0	10.0	32, 40, 50
21	1000.0	100.0	80, 100, 125
22	1000.0	10.0	32, 40, 50
23	1000.0	100.0	80, 100, 125
24	1000.0	10.0	32, 40, 50
25	1000.0	110.0	80, 100, 125

Table 10

Initial Linear Program for Problem - 1

Begin Node	End Node	Number of sections connected between the nodes
		Pressure Equations
1	6	1, 5, 8
1	7	1, 2, 3, 6
		Loop Equations
2	2	2, 3, -4, -5
4	4	4, 6, -7, -8

Table 11

Initial Linear Program for Problem - 2

Begin Node	End Node	Numbers of sections connected between the nodes
		Pressure Equations
1	5	1, 5
1	8	1, 2, 3, 11
1	9	1, 2, 6, 7, 9
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
6	6	8, 9, -10, -11

Table 12

Initial Linear Program for Problem - 3

Begin Node	End Node	Numbers of pipes connected between the two nodes
		Pressure Equations
1	8	1, 5, 13
1	9	1, 2, 3, 11
1	10	1, 2, 6, 7, 9
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
6	6	8, 9, -10, -11
5	5	4, 11, -12, -13

Table 13

Initial Linear Program for Problem -4

Begin Node	End Node	Numbers of pipes connected between the two nodes
		Pressure Equations
1	10	1, 5, 16
1	11	1, 2, 3, 14
1	12	1, 2, 6, 7, 12
1	9	1, 2, 6, 9, 10
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
4	4	9, 10, -11, -7
7	7	8, 12, -13, -14
6	6	4, 14, -15, -16

Table 14

Initial Linear Program for Problem - 5

Begin Node	End Node	Numbers of pipes connected between the two nodes
		Pressure Equations
1	10	1, 5, 18
1	11	1, 2, 3, 16
1	12	1, 2, 6, 7, 14
1	13	1, 2, 6, 9, 10, 12
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
4	4	9, 10, -11, -7
8	8	11, 12, -13, -14
7	7	8, 14, -15, -16
6	6	4, 16, -17, -18

Table 15

Initial Linear Program for Problem - 6

Begin Node	End Node	Numbers of pipes connected between the two nodes
		Pressure Equations
1	10	1, 5, 18
1	11	1, 2, 3, 16
1	14	1, 2, 6, 7, 14, 21
1	15	1, 2, 6, 9, 10, 12, 19
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
4	4	9, 10, -11, -7
8	8	11, 12, -13, -14
7	7	8, 14, -15, -16
6	6	4, 16, -17, -18
12	12	13, 19, -20, -21

Table 16

Initial Linear Program for Problem - 7

Begin Node	End Node	Numbers of pipes connected between the two nodes
		Pressure Equations
1	10	1, 5, 18
1	14	1, 2, 3, 16, 23
1	15	1, 2, 6, 7, 14, 21
1	16	1, 2, 6, 9, 10, 12, 19
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
4	4	9, 10, -11, -7
8	8	11, 12, -13, -14
7	7	8, 14, 15, -16
6	6	4, 16, -17, -18
12	12	13, 19, -20, -21
11	11	15, 21, -22, -23

Table 17

Initial Linear Program for Problem - 8

Begin	End Node	Numbers of pipes connected between the two nodes
		Pressure Equations
1	14	1, 5, 18, 25
1	15	1, 2, 3, 16, 23
1	16	1, 2, 6, 7, 14, 21
1	17	1, 2, 6. 9, 10, 12, 19
		Loop Equations
2	2	2, 3, -4, -5
3	3	6, 7, -8, -3
4	4	9, 10, -11, -7
6	6	4, 16, -17, -18
7	7	8, 14, -15, -16
8	8	11, 12, -13, -14
12	12	13, 19, -20, -21
11	11	15, 21, -22, -23
10	10	17, 23, -24, -25

5. RESULTS AND DISCUSSION

The optimum designs obtained from execution of the program are given in following tables.

Problem 1	-	Table 18
Problem 2	-	Table 19
Problem 3	-	Table 20
Problem 4	-	Table 21
Problem 5	-	Table 22
Problem 6	-	Table 23
Problem 7	-	Table 24
Problem 8	-	Table 25

Table 18

Optimum Design for Problem - 1

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	600.00	1000.0	100	1000.00	-	-
2	283.53	1000.0	80	1000.00	-	-
3	183.53	1000.0	80	1000.00	-	-
4	2.57	1000.0	15	1000.00	-	-
5	216.47	1000.0	80	799.05	63	200.95
6	86.10	1000.0	63	1000.00	-	-
7	13.90	1000.0	32	1000.00	-	-
8	113.90	1000.0	63	994.09	50	5.91

Optimum pumping head at Node 1 = 35 + 18.26 = 53.26 m

Minimum cost of the system = 11898.25 Rs/year

Minimum cost for the initial flow pattern = 12093.24 Rs/year

At pipe no. 2, no booster pump is required.

Table 19

Optimum Design for Problem - 2

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	800.00	1000.0	125	1000.00	-	-
2	599.99	1000.0	125	565.94	100	434.06
3	211.77	1000.0	80	996.64	63	3.36
4	0.01	1000.0	15	1000.00	-	-
5	100.00	1000.0	50	1000.00	-	-
6	288.22	1000.0	100	1000.00	-	-
7	188.22	1000.0	80	1000.00	-	-
8	11.50	1000.0	32	1000.00	-	-
9	99.72	1000.0	80	667.62	63	322.38
10	0.27	1000.0	15	1000.00	-	-
11	100.27	1000.0	63	1000.00	-	-

Optimum pumping head at node 1 = 35 + 12.6 = 47.6 m

Minimum cost of the system = 18238.60 Rs/year

Minimum cost for the initial flow pattern = 18842.00 Rs/year

At pipe no. 6, no booster pump is required.

Table 20

Optimal Design for Problem - 3

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	900.00	1000.00	125	1000.00	-	-
2	584.18	1000.00	125	304.82	100	695.18
3	199.94	1000.00	80	1000.00	-	-
4	2.18	1000.00	15	1000.00	-	-
5	215.81	1000.00	80	1000.00	-	-
6	280.06	1000.00	100	992.55	80	7.45
7	180.06	1000.00	80	1000.00	-	-
8	10.73	1000.00	32	913.37	25	86.63
9	90.79	1000.00	63	1000.00	-	-
10	9.20	1000.00	32	1000.00	-	-
11	95.58	1000.00	63	941.89	50	58.11
12	13.63	1000.00	32	1000.00	-	-
13	113.63	1000.00	63	1000.00	-	-

Optimum pumping head at node 1 = 35 + 18.22 = 53.22 m

Minimum cost of the system = 21417.43 Rs/year

Minimum cost for the initial flow distribution = 21648.22 Rs/year

No booster pump is required at pipe No. 6.

Table 22

Optimum Design for Problem - 5

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	1200.00	10000.0	150	1000.00	-	-
2	883.98	1000.0	150	999.03	125	0.97
3	199.59	1000.0	80	877.46	63	122.54
4	2.28	1000.0	20	44.59	15	955.41
5	216.02	1000.0	80	1000.00	-	-
6	584.39	1000.0	125	1000.00	-	-
7	204.30	1000.0	80	1000.00	-	-
8	1.87	1000.0	15	1000.00	-	-
9	280.09	1000.0	100	999.47	80	0.53
10	180.09	1000.0	80	1000.00	-	-
11	11.15	1000.0	32	1000.00	-	-
12	91.24	1000.0	80	14.01	63	985.99
13	8.76	1000.0	32	1000.00	-	-
14	95.02	1000.0	63	950.11	50	49.89
15	13.74	1000.0	32	1000.00	-	-
16	100.00	1000.0	63	1000.00	-	-
17	13.74	1000.0	32	1000.00	-	-
18	113.74	1000.0	63	1000.00	-	-

Optimum pumping head at node 1 = 35 + 12.14 = 47.14

Optimum cost of the system = 31411.34 Rs/year

Optimum cost of the initial flow pattern = 31852.17 Rs/year

No booster pump is required at pipe no. 6.

Table 23

Optimum Design for Problem - 6

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	1400.00	1000.0	150	1000.00	-	-
2	1084.71	1000.0	150	1000.00	-	-
3	199.18	1000.0	80	684.31	63	351.69
4	2.03	1000.0	15	1000.00	-	-
5	215.29	1000.0	80	611.21	63	388.79
6	785.53	1000.0	125	1000.00	-	-
7	312.06	1000.0	100	1000.00	-	-
8	1.58	1000.0	15	1000.00	-	-
9	373.47	1000.00	100	1000.00	-	-
10	273.47	1000.0	100	977.19	80	22.81
11	5.05	1000.0	25	9.69	20	990.31
12	178.52	1000.0	80	1000.00	-	-
13	14.38	1000.0	32	1000.00	-	-
14	208.59	1000.0	80	1000.00	-	-
15	12.89	1000.0	32	1000.00	-	-
16	99.63	1000.0	63	944.45	50	55.55
17	13.26	1000.0	32	1000.00	-	-
18	113.26	1000.0	63	998.78	50	1.22
19	92.90	1000.0	80	1000.00	-	-
20	7.10	1000.0	32	1000.00	-	-
21	107.10	1000.0	80	407.77	63	592,23

Optimum pumping head at node 1 = 35 + 24.40 = 59.40 m

Minimum cost of the system = 40174.68 Rs/year

Minimum cost for the initial flow pattern = 40813.91 Rs/year

No booster pump is required at pipe no. 6.

Table 24

Optimum Design for Problem - 7

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	1500.00	1000.0	150	1000.00	-	-
2	1184.56	1000.0	150	1000.00	-	-
3	308.49	1000.0	100	499.32	80	500.68
4	2.80	1000.0	15	1000.00	-	-
5	215.44	1000.0	80	879.58	63	120.42
6	776.07	1000.0	125	1000.00	-	-
7	300.06	1000.0	100	962.91	80	37.09
8	2.01	1000.0	15	1000.00	-	-
9	376.01	1000.0	100	1000.00	-	-
10	276.01	1000.0	100	1000.00	-	-
11	2.32	1000.0	20	34.09	15	965.91
12	178.33	1000.0	80	1000.00	-	-
13	15.34	1000.0	32	1000.00	-	-
14	199.75	1000.0	80	1000.00	-	-
15	11.32	1000.0	32	837.04	25	162.96
16	209.28	1000.0	80	1000.00	-	-
17	12.64	1000.0	32	1000.00	-	-
18	112.64	1000.0	63	667.17	50	332.83
19	93.67	1000.0	80	1000.00	-	-
20	6.32	1000.0	32	1000.00	-	-
21	95.72	1000.0	63	984.96	50	150.34
22	10.60	1000.0	32	1000.00	-	-
23	110.60	1000.0	80	23.21	63	976.79

Optimum pumping head at node 1 = 35 + 26.89 = 61.89 m

Minimum cost of the system = 43644.54 Rs/year

Minimum cost for the initial flow distribution = 44399.74 Rs/year

No booster pump is required at pipe no. 6.

Table 25

Optimum Results for Problem - 8

Pipe No.	Flow (lpm)	Length (m)	Segment 1		Segment 2	
			Diameter (mm)	Length (m)	Diameter (mm)	Length (m)
1	1600.00	1000.0	150	1000.00	-	-
2	1177.45	1000.0	150	1000.00	-	-
3	307.09	1000.0	80	1000.00	-	-
4	2.53	1000.0	15	1000.00	-	-
5	322.54	1000.0	100	15.45	80	984.55
6	770.36	1000.0	125	1000.00	-	-
7	292.13	1000.0	100	556.24	80	443.76
8	9.43	1000.0	20	947.68	15	52.32
9	378.23	1000.0	100	1000.00	-	-
10	278.23	1000.0	100	786.84	80	213.16
11	2.03	1000.0	15	1000.00	-	-
12	180.26	1000.0	80	1000.00	-	-
13	1.77	1000.0	20	34.20	15	965.80
14	199.53	1000.0	80	1000.00	-	-
15	11.50	1000.0	32	1000.00	-	-
16	200.19	1000.0	80	1000.00	-	-
17	2.82	1000.0	20	435.61	15	564.38
18	220.01	1000.0	80	1000.00	-	-
19	82.04	1000.0	63	1000.00	-	-
20	17.96	1000.0	50	1000.00	-	-
21	109.26	1000.0	80	11.61	63	988.39
22	8.70	1000.0	32	1000.00	-	-
23	91.51	1000.0	63	705.89	50	294.10
24	17.19	1000.0	40	43.33	32	956.67
25	117.19	1000.0	63	1000.00	-	-

Optimum pumping head at node 1 = 35 + 32.51 = 67.51 m

Minimum cost of the system = 46819.07 Rs/year

Minimum cost for initial flow distribution = 47581.68 Rs/year

No booster pump is required at pipe no. 9.

As mentioned earlier also, 15 mm dia pipe should not have been included in the list of available pipes but as it was included so the program in its attempt to minimize the objective function has provided 15 mm pipes for some minor pipe connections.

The program gives the lengths of pipe segments of adjacent available diameters to be provided between any two nodes in odd figures. These should be rounded off to the nearest suitable values. Then a network solver should be run to check for the heads available at the nodes. This has not been done in the present case and the pipe lengths have been reported upto two decimal places (These have to be rounded off, if possible, to get the multiples of a preassigned value, say 1 meter).

To check if the program worked well with respect to the pressure constraints and loop constraints, the head losses in pipes and pressures at nodes are computed for Problem 1 and are reported in Table 26. The pressure constraints and loop constraints are found to be satisfied.

The program was executed for the 8 problems given in the preceding section. The execution time along with the size of problem is given in Table 27. The increase in Central Processing Unit (CPU) time with the increasing size of the problem is shown in Figure 10.

Table 26

Head Losses in Pipes and Available Heads at Nodes for Problem - 1

Pipe No.	Head loss (m)		Node No.	Head available (m)
	Segment 1	Segment 2		
1	16.417	-	1	53.26
2	12.143	-	2	36.844
3	5.426	-	3	24.701
4	6.944	-	4	26.219
5	5.886	4.738	5	19.275
6	4.275	-	6	18.953
7	3.953	-	7	15.000
8	7.136	0.131		

Table 27

Variation of Computer Time With Size of Problems Taken Up

Problem No.	CPU time (seconds)	No. of flow iteration	CPU time per iteration (seconds)	No. of decision variables	No. of constraints
1	3.08	11	0.28	39	12
2	19.31	24	0.804	53	17
3	9.71	9	1.078	62	20
4	33.92	16	2.12	76	25
5	54.23	18	3.013	85	28
6	68.13	16	4.258	98	32
7	121.54	20	6.077	107	35
8	81.59	11	7.414	116	38

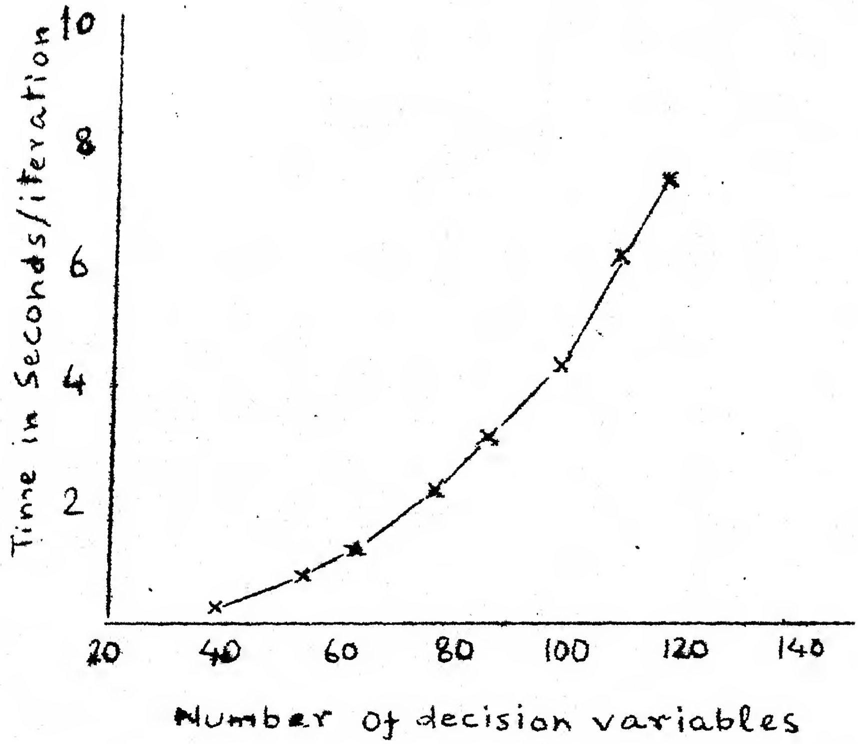
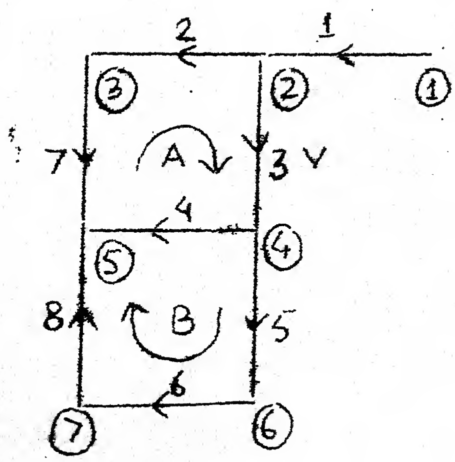


FIG-10



Node numbers are in circles.
Pipe numbers are shown along the pipes.

FIG - 11

6. CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

6.1 Conclusions

The program was found to work well for the problems framed in Section 4. The CPU time increases rapidly with the increasing size of the problem. The salient features of the program are as follows:

(a) The program does not require a network solver. It incorporates the flow solution into the optimization procedure, without making any assumptions about the hydraulic solution of the network.

(b) Operational decisions are included explicitly in the design process.

(c) The method yields a design that is hydraulically feasible and is closer to being optimal than the one from which the search is started; this holds true even when the optimization procedure is terminated prematurely.

(d) Algorithm can be used to satisfy exact flow requirements along the pipes if required.

Some weak points of the algorithm are:

(a) Computer memory required is quite large as every pipe requires three decision variables in the LPG method.

(b) Flows into and out of the reservoirs have to be fixed beforehand. This ensures proper operation of the

reservoir but does not include their capacity as a decision variable.

6.2 Suggestions for Further Work

(1) The algorithm can be made more efficient by incorporating some features as given below:

a) The candidate list of diameters for any pipe is extended on the higher or lower side according to the requirement and the problem is reset and solved again. However if the number of diameter changes required is not significant compared to total number of decision variables, the post-optimality analysis can be used to get the solution using the solutions of the last problem.

b) Similarly when flows in the pipes are changed, the post-optimality considerations can be employed to save computer time.

c) A unidirectional search method can be suitably used to determine the step length while deciding about the flow change.

(2) The program can be made to incorporate consideration of more than one demand pattern as proposed by Alperovits and Shamir[2] by introducing 'artificial' valves in each loop.

(3) The program can be used for optimal expansion of an existing system or to decide about the operating

policies of an existing system as outlined by Alperovits and Shamir[2].

(4) There is no provision in the algorithm to reverse the flow directions in some pipes if required to minimize the cost. Hence some criterion should be developed to come out with a few suitable flow patterns and the one which gives the minimum cost should be taken as the optimum design.

(5) As computer memory required is quite large for large water distribution systems hence the application of sparce matrix techniques can save a considerable computer memory and time.

(6) More efficient algorithms for network optimization may be tried (e.g. Out of Kilter Algorithm).

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APPENDIX 1

DERIVATION OF GRADIENT EXPRESSION

For illustrative purpose the gradient expression given in general farm equation (19), is derived for the example problem shown in Figure 11. For simplicity only the two loop paths are considered. It is assumed that additional paths, which would be used to specify minimum node pressures, are not included, and therefore such paths are not considered in the derivation of the gradient expressions. New notation is defined as follows:

Hl_{ij} = head loss in the links between nodes i and j in the direction of the flow;

Q_{ij} = flow in the link between i and j;

$d\alpha$ = change in flow in all the links in loop A;

$d\beta$ = change in flow in all the links in loop B;

y = head discontinuity at node 4 from loop A;

δ = head discontinuity at node 5 from loop B.

The loop constraints from the linear program can be written as:

$$Hl_{45} - Hl_{35} - Hl_{23} + Hl_{24} = 0 \tag{50}$$

$$-Hl_{45} + Hl_{46} + Hl_{67} + Hl_{75} = 0 \tag{51}$$

The dual variables for these constraints, W_A and W_B, respectively, represent the change in cost, that would result if the head loss in the loops could change. If a

discontinuity in head loss is introduced, W_A and W_B can be used to determine whether the discontinuities in the right hand sides of the constraints should be positive or negative to reduce the cost of the system. In the physical system, changes in the flows in the links are necessary to balance the head losses (i.e., to 'heal' the discontinuity). Since flow and head losses are not independent, the necessary changes in flows can be predicted.

If the change in flow in each link is noted as dQ_{ij},

$$dQ_{23} = -d\alpha \quad (52)$$

$$dQ_{24} = d\alpha \quad (53)$$

$$dQ_{45} = d\alpha - d\beta \quad (54)$$

$$dQ_{46} = d\beta \quad (55)$$

$$dQ_{67} = d\beta \quad (56)$$

$$dQ_{35} = -d\alpha \quad (57)$$

$$dQ_{75} = d\beta \quad (58)$$

The discontinuity expressions are written explicitly as

$$Hl_{45} - Hl_{35} - Hl_{23} + Hl_{24} = y \quad (59)$$

$$-Hl_{45} + Hl_{46} + Hl_{67} + Hl_{75} = \delta \quad (60)$$

The total change in head loss around the loops due to changes in flows should be $-y$ and $-\delta$.

$$\frac{\partial H1_{45}}{\partial Q_{45}} dQ_{45} - \frac{\partial H1_{35}}{\partial Q_{35}} dQ_{35} - \frac{\partial H1_{23}}{\partial Q_{23}} dQ_{23} + \frac{\partial H1_{24}}{\partial Q_{24}} dQ_{24}$$

$$- \frac{\partial H1_{45}}{\partial Q_{45}} dQ_{45} + \frac{\partial H1_{46}}{\partial Q_{46}} dQ_{46} + \frac{\partial H1_{67}}{\partial Q_{67}} dQ_{67} + \frac{\partial H1_{75}}{\partial Q_{75}} dQ_{75}$$

Substituting for dQ_{ij} from equations (52) to (58)

$$\frac{\partial H1_{45}}{\partial Q_{45}} (d\alpha - d\beta) - \frac{\partial H1_{35}}{\partial Q_{35}} (- d\alpha) - \frac{\partial H1_{23}}{\partial Q_{23}} (- d\alpha)$$

$$+ \frac{\partial H1_{24}}{\partial Q_{24}} d\alpha + dy = 0 \tag{63}$$

$$- \frac{H1_{45}}{Q_{45}} (d\alpha - d\beta) + \frac{\partial H1_{46}}{\partial Q_{46}} d\beta + \frac{\partial H1_{67}}{\partial Q_{67}} d\beta$$

$$+ \frac{\partial H1_{75}}{\partial Q_{75}} d\beta + d\delta = 0 \tag{64}$$

All the information required to determine the change in cost with respect to a change in the flow in the paths is now available,

$$\frac{\partial(\text{Cost})}{\partial \alpha} = \frac{\partial(\text{Cost})}{\partial y} \cdot \frac{\partial y}{\partial \alpha} + \frac{\partial(\text{Cost})}{\partial \delta} \cdot \frac{\partial \delta}{\partial \alpha} \tag{65}$$

$$\frac{\partial(\text{Cost})}{\partial \beta} = \frac{\partial(\text{Cost})}{\partial y} \cdot \frac{\partial y}{\partial \beta} + \frac{\partial(\text{Cost})}{\partial \delta} \cdot \frac{\partial \delta}{\partial \beta} \tag{66}$$

The terms $\partial(\text{Cost})/\partial y$ and $\partial(\text{Cost})/\partial\delta$ are the dual variables for the loops, and the remaining terms can be found by picking out the coefficients of the differentials in equations (63) and (64).

$$\frac{\partial \gamma}{\partial \alpha} = -\frac{\partial Hl_{45}}{\partial Q_{45}} - \frac{\partial Hl_{35}}{\partial Q_{35}} - \frac{\partial Hl_{23}}{\partial Q_{23}} - \frac{\partial Hl_{24}}{\partial Q_{24}} \qquad (67)$$

$$\frac{\partial \gamma}{\partial \beta} = +\frac{\partial Hl_{45}}{\partial Q_{45}} \qquad (68)$$

$$\frac{\partial \delta}{\partial \alpha} = \frac{\partial Hl_{45}}{\partial Q_{45}} \qquad (69)$$

$$\frac{\partial \delta}{\partial \beta} = -\frac{\partial Hl_{45}}{\partial Q_{45}} - \frac{\partial Hl_{46}}{\partial Q_{46}} - \frac{\partial Hl_{67}}{\partial Q_{67}} - \frac{\partial Hl_{75}}{\partial Q_{75}} \qquad (70)$$

Substituting these partial derivatives and the dual variables into equations (65) and (66), and using equation ((1) for $\frac{\partial Hl_{ij}}{\partial Q_{ij}}$, we get

$$G_A = \frac{\partial(\text{Cost})}{\partial \alpha} = -W_A\left(\frac{Hl_{45}}{Q_{45}} + \frac{Hl_{35}}{Q_{35}} + \frac{Hl_{23}}{Q_{23}} + \frac{Hl_{24}}{Q_{24}}\right) + W_B\left(\frac{Hl_{45}}{Q_{45}}\right) \qquad (71)$$

$$G_B = \frac{\partial(\text{Cost})}{\partial \beta} = W_A\left(\frac{Hl_{45}}{Q_{45}}\right) - W_B\left(\frac{Hl_{45}}{Q_{45}} + \frac{Hl_{46}}{Q_{46}} + \frac{Hl_{67}}{Q_{67}} + \frac{Hl_{75}}{Q_{75}}\right) \qquad (72)$$

A generalization of the above expression leads to the gradient equation given previously as (19).

APPENDIX 2

NOTATIONS

Q	=	Discharge from a pipe
C_{HW}	=	Hazen-Williams coefficient
D	=	Diameter of pipe
H	=	Head loss between the ends of a pipe
L	=	Length of pipe
J	=	Hydraulic gradient
a, b, c	=	Numerical coefficients
i, j	=	Nodes
m	=	Number of pipe diameter in the candidate list of diameters
C	=	Cost factor for the decision variable
I	=	Inflow
X	=	Decision variable